**Student Notes**

## Student Study Session Topic: Interpreting Graphs

Starting with the graph of a function or its derivative you may be asked all kinds of questions without having (or needing) and equation to work with. Working from a graph you may be asked for the location of extreme values, intervals where the function is increasing or decreasing, points of inflection, concavity, the value of a definite integral, etc.

In each case you are given a graph, or can use your graphing calculator to sketch one. You are expected to answer the questions from the graph alone. Even if you can write the equation of the functions from the graph (and often you can), you should not do so. What you need to know is on the graph and using the graph is the most efficient way to answer the questions.

**What you should know how to do:**

- Read information about the *function* from the graph of its *derivative*. This may be approached as a derivative techniques or antiderivative techniques.
- Find and justify local and absolute extreme values ($1^{st}$ derivative test, $2^{nd}$ derivative test).
- Find and justify points of inflection.
- Write an equation of tangent line (read point and slope from graph).
- Evaluate definite integrals (FTC) from graph areas.
- Work with functions defined by integrals.

**Prep session practice.** As instructed by the session presenter work on the following problems.

**2006 AB 3:** A typical problem. You are given the graph of the derivative and asked questions about the function.

**2003 AB4/BC4:** A similar problem asking some different things.

**2008 AB 4/BC4**: This is a particle motion problem, but it has a lot of similarities with the two preceding problems. You are given the graph of the velocity (= the first derivative) and asked about the particle's position (= the antiderivative or the accumulated velocity = distance).

**Multiple-choice questions** also test these same ideas. Several questions of this type appear on every AP Calculus exam.

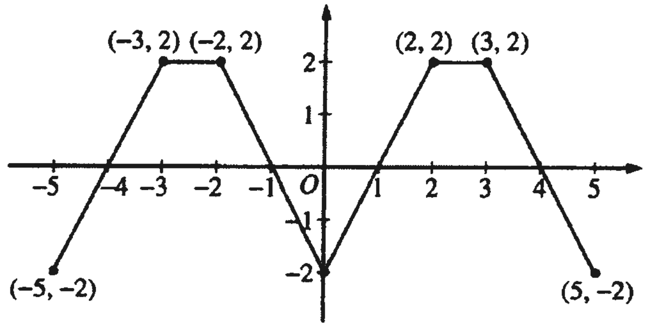
<u>Selection</u>: 1, 2, 5*, 6*,7, 8, 9, 11, 13, 16 (* 5 and 6 are graphing calculator questions)

**WATCH and LISTEN to the multiple-choice questions being solved**

**Go to http://tinyurl.com/NMSI-Math-6 Click on the "Full Screen" arrow. Then click anywhere on the page to see and hear from that point on.**

**Click anywhere to go back anytime.**

Answers: 1 d, 2 a, 3 b, 4 b, 5* b, 6* b, 7 c, 8 e, 9 a, 10 a, 11 d, 12 c, 13 b, 14 a, 15 a, 16 e
(* 5 and 6 are graphing calculator questions)




Graph of $f$

3. The graph of the function $f$ shown above consists of six line segments. Let $g$ be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(4)$, $g'(4)$, and $g''(4)$.

   (b) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 1$ ? Justify your answer.

   (c) Suppose that $f$ is defined for all real numbers $x$ and is periodic with a period of length 5. The graph above shows two periods of $f$. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of $g$ at $x = 108$.

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**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

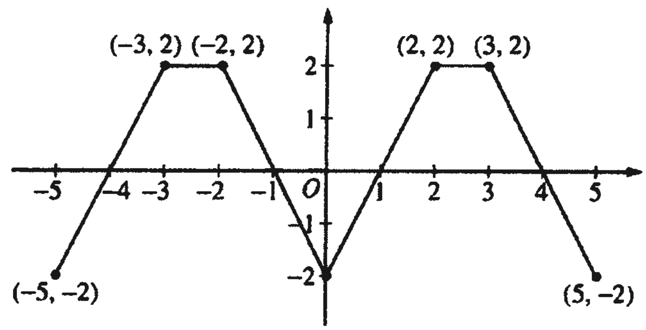
**END OF PART A OF SECTION II**

## Question 3

The graph of the function $f$ shown above consists of six line segments. Let $g$ be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does $g$ have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.




Graph of $f$

(c) Suppose that $f$ is defined for all real numbers $x$ and is periodic with a period of length 5. The graph above shows two periods of $f$. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of $g$ at $x = 108$.

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(a) $g(4) = \int_0^4 f(t)\,dt = 3$

$g'(4) = f(4) = 0$

$g''(4) = f'(4) = -2$

$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$

(b) $g$ has a relative minimum at $x = 1$ because $g' = f$ changes from negative to positive at $x = 1$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(c) $g(0) = 0$ and the function values of $g$ increase by 2 for every increase of 5 in $x$.

$g(10) = 2g(5) = 4$

$$\begin{aligned} g(108) &= \int_0^{105} f(t)\,dt + \int_{105}^{108} f(t)\,dt \\ &= 21g(5) + g(3) = 44 \end{aligned}$$

$g'(108) = f(108) = f(3) = 2$

An equation for the line tangent to the graph of $g$ at $x = 108$ is $y - 44 = 2(x - 108)$.

$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$

**CALCULUS AB**
**SECTION II, Part B**
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**

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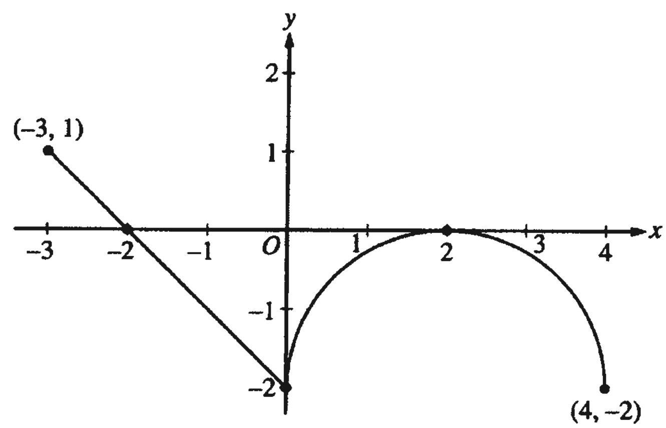


Graph of $f'$

4. Let $f$ be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.
   (a) On what intervals, if any, is $f$ increasing? Justify your answer.
   (b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.
   (c) Find an equation for the line tangent to the graph of $f$ at the point $(0, 3)$.
   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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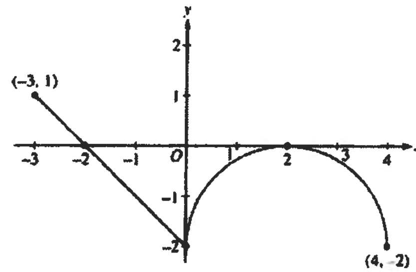
**GO ON TO THE NEXT PAGE.**

## Question 4

Let $f$ be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'$, the derivative of $f$, consists of one line segment and a semicircle, as shown above.




Graph of $f'$

(a) On what intervals, if any, is $f$ increasing? Justify your answer.

(b) Find the $x$-coordinate of each point of inflection of the graph of $f$ on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of $f$ at the point $(0,3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

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(a) The function $f$ is increasing on $[-3,-2]$ since $f' > 0$ for $-3 \le x < -2$.

$2: \begin{cases} 1: \text{interval} \\ 1: \text{reason} \end{cases}$

(b) $x = 0$ and $x = 2$

$f'$ changes from decreasing to increasing at $x = 0$ and from increasing to decreasing at $x = 2$

$2: \begin{cases} 1: x = 0 \text{ and } x = 2 \text{ only} \\ 1: \text{justification} \end{cases}$

(c) $f'(0) = -2$

Tangent line is $y = -2x + 3$.

1 : equation

(d) $$f(0) - f(-3) = \int_{-3}^{0} f'(t)\,dt = \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2}$$

$$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$$

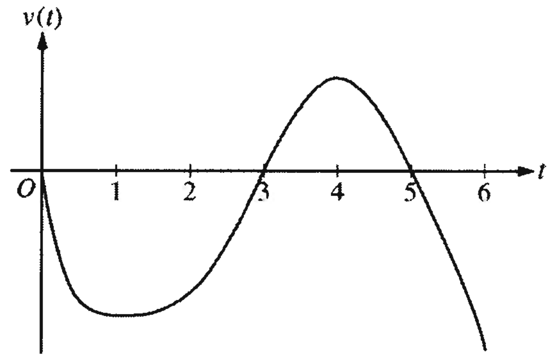
$$f(4) - f(0) = \int_{0}^{4} f'(t)\,dt = -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi$$

$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$

4 :
- 1 : $\pm\left(\frac{1}{2} - 2\right)$ (difference of areas of triangles)
- 1 : answer for $f(-3)$ using FTC
- 1 : $\pm\left(8 - \frac{1}{2}(2)^2\pi\right)$ (area of rectangle − area of semicircle)
- 1 : answer for $f(4)$ using FTC

**CALCULUS AB**
**SECTION II, Part B**
**Time—45 minutes**
**Number of problems—3**

**No calculator is allowed for these problems.**




Graph of $v$

4. A particle moves along the $x$-axis so that its velocity at time $t$, for $0 \le t \le 6$, is given by a differentiable function $v$ whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the $t$-axis and the graph of $v$ on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of $t$, where $0 \le t \le 6$, is the particle at $x = -8$ ? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

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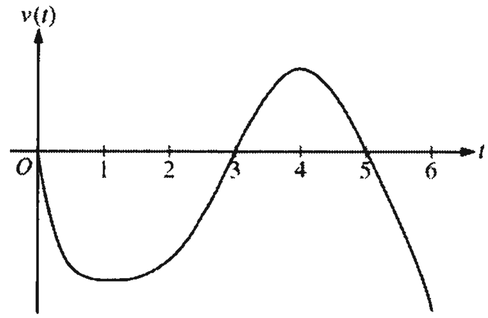
**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

## Question 4




Graph of $v$

A particle moves along the $x$-axis so that its velocity at time $t$, for $0 \le t \le 6$, is given by a differentiable function $v$ whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the $t$-axis and the graph of $v$ on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of $t$, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

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| | |
|---|---|
| (a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$.<br>$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$<br>$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$<br>Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$. | $3 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{considers } \int_0^6 v(t)\,dt \\ 1 : \text{conclusion} \end{cases}$ |
| (b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$.<br>By the Intermediate Value Theorem, there are three values of $t$ for which the particle is at $x(t) = -8$. | $3 : \begin{cases} 1 : \text{positions at } t = 3,\ t = 5, \\ \quad\ \text{and } t = 6 \\ 1 : \text{description of motion} \\ 1 : \text{conclusion} \end{cases}$ |
| (c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and $v$ is increasing. | 1 : answer with reason |
| (d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals. | $2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$ |

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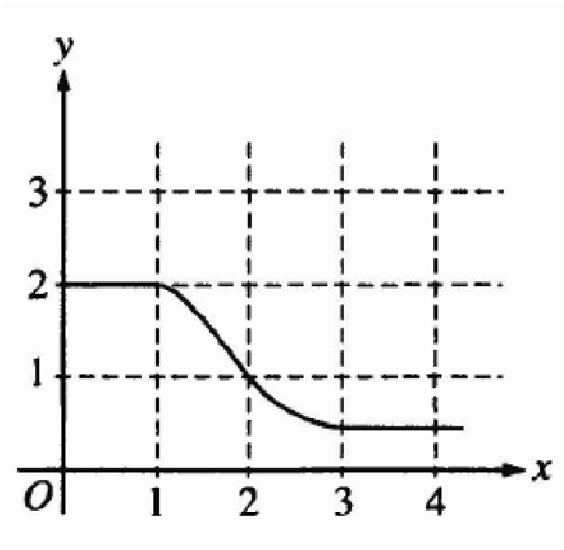
# Interpreting Graphs

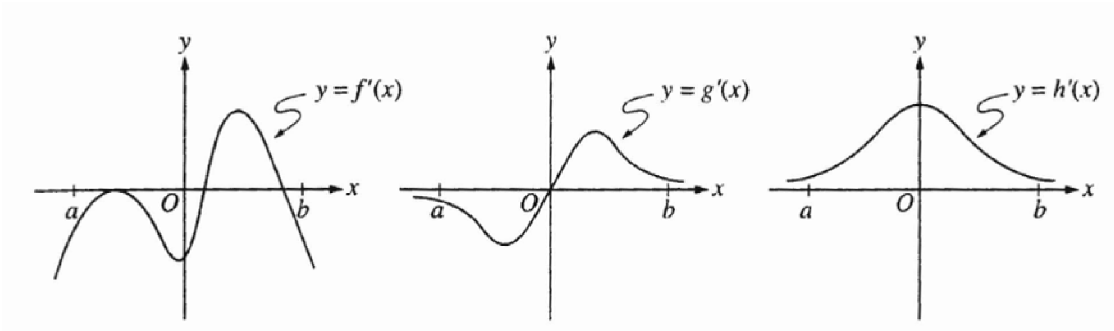
**Multiple Choice**
*Identify the choice that best completes the statement or answers the question.*

1.



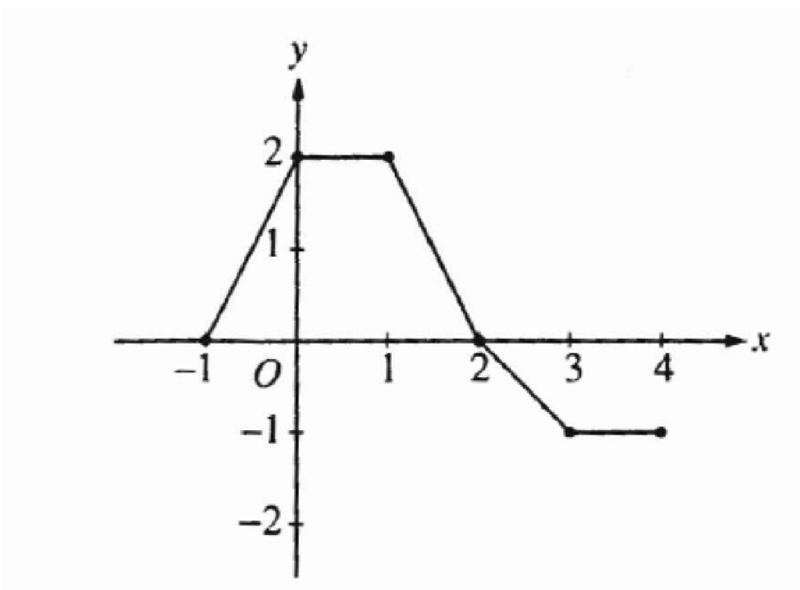

The graph of *f* is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

a. 0.3
b. 1.3
c. 3.3
d. 4.3
e. 5.3

2.


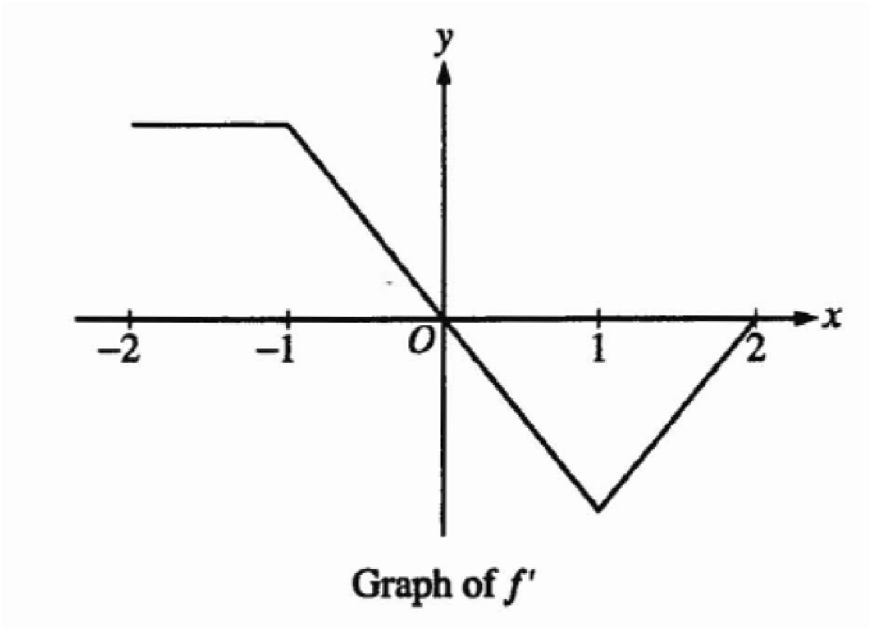

The graphs of the derivatives of the functions *f*, *g*, and *h* are shown above. Which of the functions *f*, *g*, or *h* have a relative maximum on the open interval $a < x < b$ ?

a. *f* only
b. *g* only
c. *h* only
d. *f* and *g* only
e. *f*, *g*, and *h*

3.



The graph of a piecewise-linear function *f*. for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

a. 1
b. 2.5
c. 4
d. 5.5
e. 8

4. 


The graph of $f'$, the derivative of the function $f$, is shown above. Which of the following statements is true about $f$?

a. $f$ is decreasing for $-1 \le x \le 1$.
b. $f$ is increasing for $-2 \le x \le 0$.
c. $f$ is increasing for $1 \le x \le 2$.
d. $f$ has a local minimum at $x = 0$.
e. $f$ is not differentiable at $x = -1$ and $x = 1$.

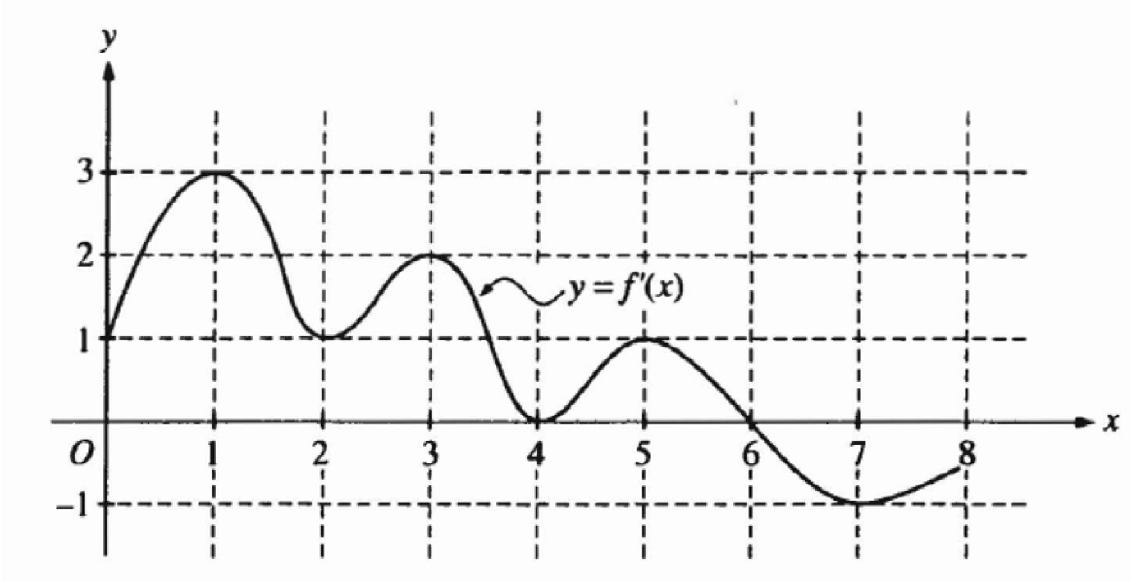
5. The first derivative of the function $f$ is given by $f'(x) = \dfrac{\cos^2 x}{x} - \dfrac{1}{5}$. How many critical values does $f$ have on the open interval (0, 10)?

a. One
b. Three
c. Four
d. Five
e. Seven

6. The function $f$ has fIrst derivative given by $f'(x) = \dfrac{\sqrt{x}}{1 + x + x^3}$ What is the x-coordinate ofthe inflection point of the graph of $f$?

a. 1.008
b. 0.473
c. 0
d. -0.278
e. The graph of $f$ has no inflection point.

Questions 7-9 refer to the graph and the information given below.




The function $f$ is defined on the closed interval [0, 8]. The graph of its derivative $f'$ is shown above.

7. The point (3, 5) is on the graph of $y = f(x)$. An equation of the line tangent to the graph of $f$ at (3, 5) is

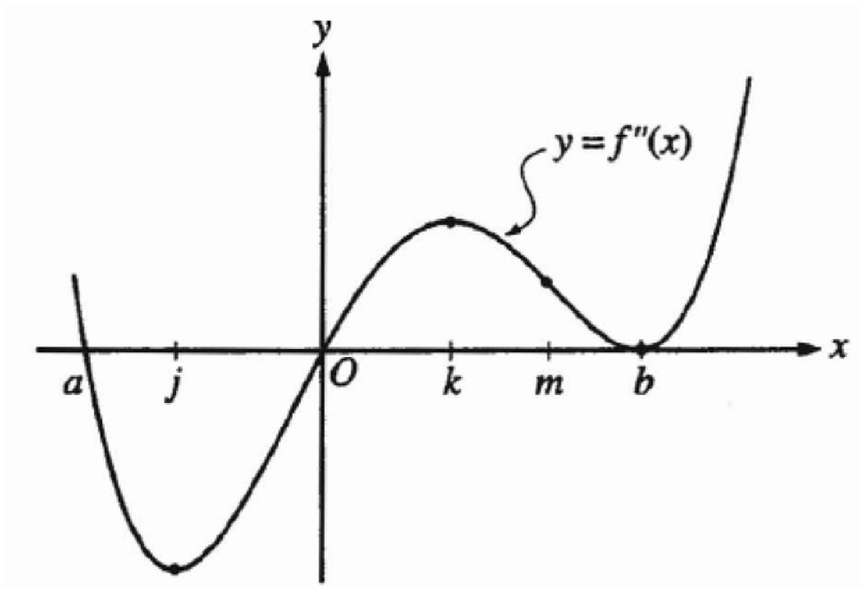
   a. $y = 2$
   b. $y = 5$
   c. $y - 5 = 2(x - 3)$
   d. $y + 5 = 2(x - 3)$
   e. $y + 5 = 2(x + 3)$

8. How many points of inflection does the graph of $f$ have?

   a. Two
   b. Three
   c. Four
   d. Five
   e. Six

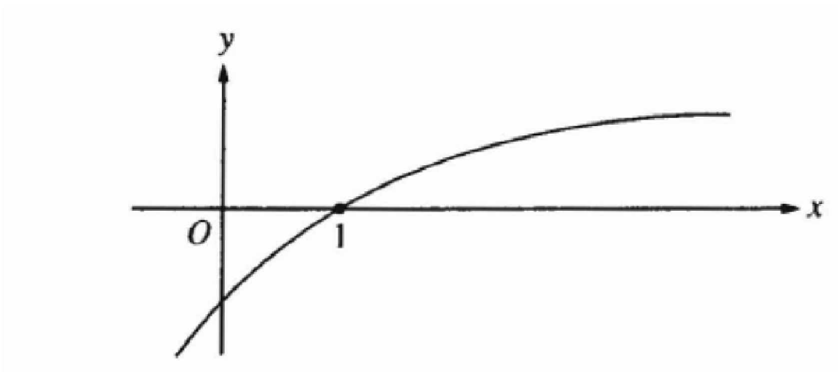
9. At what value of $x$ does the absolute minimum of $f$ occur?

   a. 0
   b. 2
   c. 4
   d. 6
   e. 8

10. 


The second derivative of the function *f* is given by $f''(x) = x(x-a)(x-b)^2$ The graph of $f''$ is shown above. For what values of *x* does the graph of *f* have a point of inflection?

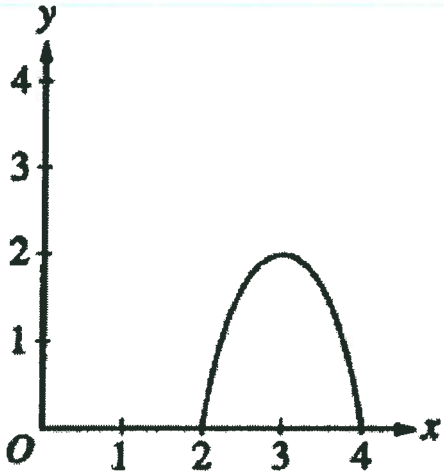
a. 0 and *a* only
b. 0 and *m* only
c. *b* and *j* only
d. 0, *a*, and *b*
e. *b*. *j*, and *k*

11. 

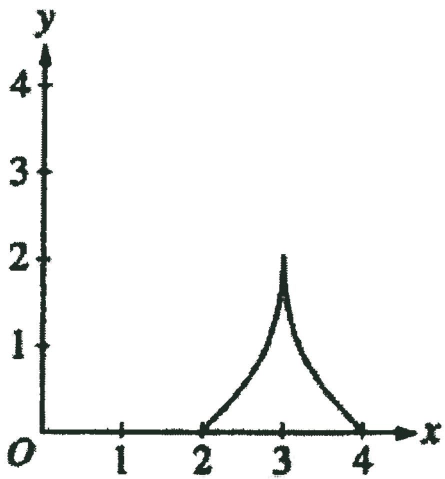

The graph of a twice-differentiable function *f* is shown In the figure above. Which of the following is true?

a. $f(1) < f'(1) < f''(1)$
b. $f(1) < f''(1) < f'(1)$
c. $f'(1) < f(1) < f''(1)$
d. $f''(1) < f(1) < f'(1)$
e. $f''(1) < f'(1) < f(1)$

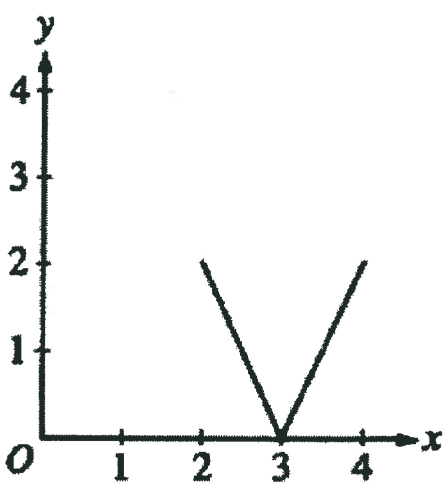
12. On the closed interval [2, 4], which of the following could be the graph of a function $f$ with the property that $\frac{1}{4-2}\int_{2}^{4} f(t)\,dt = 1$?



a.



b.



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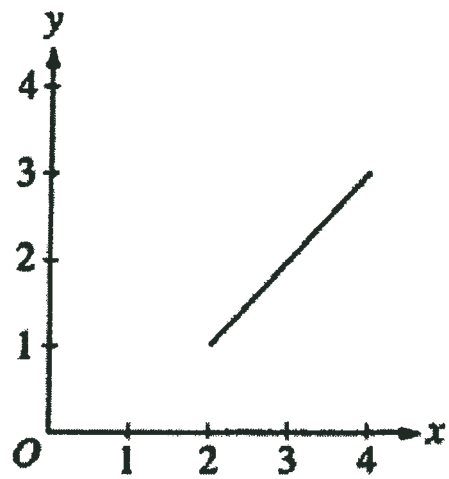
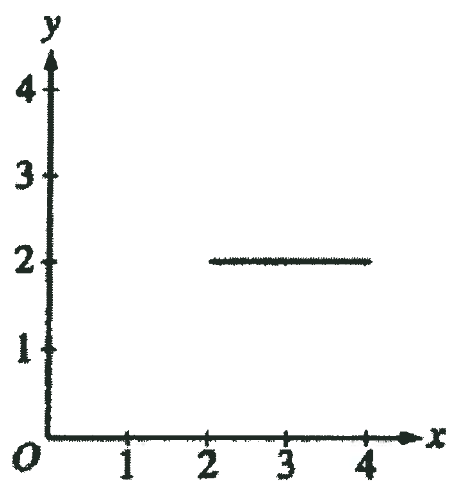
c.



d.



e.

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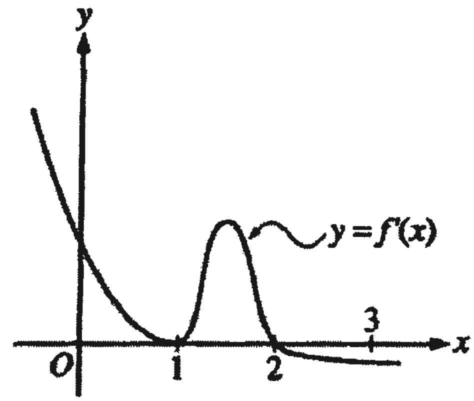
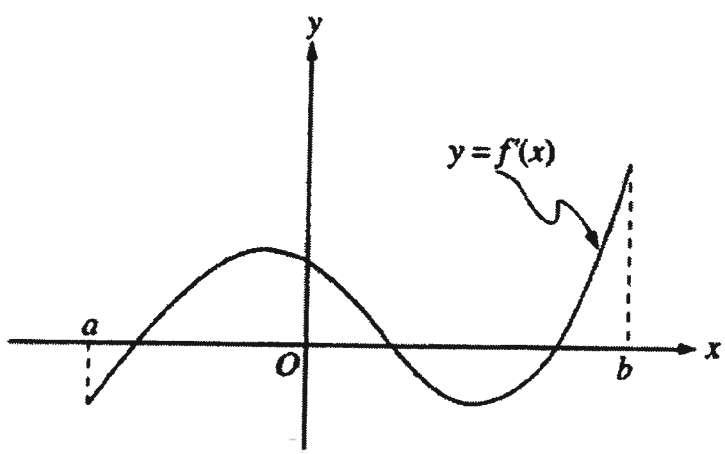
13.




The graph of $f'$, the derivative of the function $f$, is shown above. If $f(0) = 0$, which of the following must be true?

I. $f(0) > f(1)$
II. $f(2) > f(1)$
III. $f(1) > f(3)$

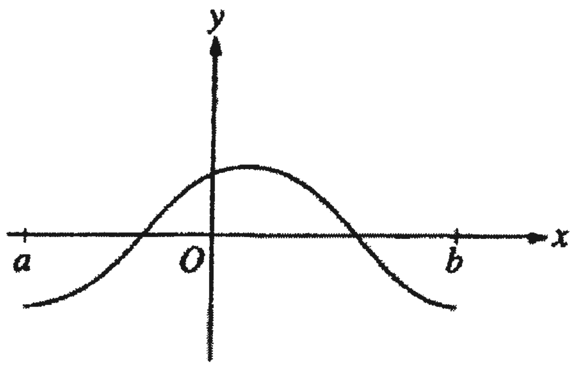
a. I only
b. II only
c. III only
d. I and II only
e. II and III only




14.

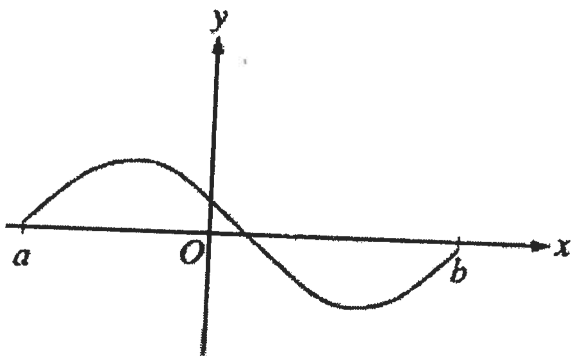
The graph of $f'$, the derivative of $f$, is shown in the figure above. Which of the following describes all relative extrema of $f$ on the open interval $(a, b)$ ?

a. One relative maximum and two relative minima
b. Two relative maxima and one relative minimum
c. Three relative maxima and one relative minimum
d. One relative maximum and three relative minima
e. Three relative maxima and two relative minima

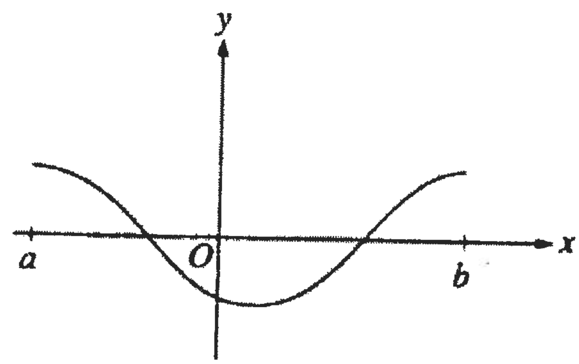



15.

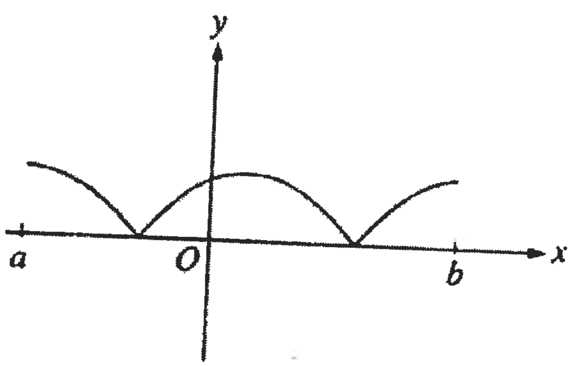
The graph of *f* is shown in the figure above. Which of the following could be the graph of the derivative of *f*?



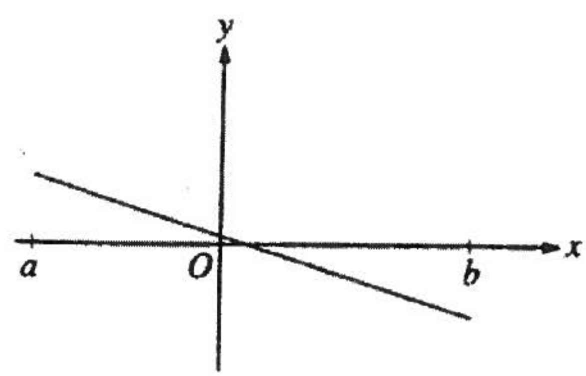

a.



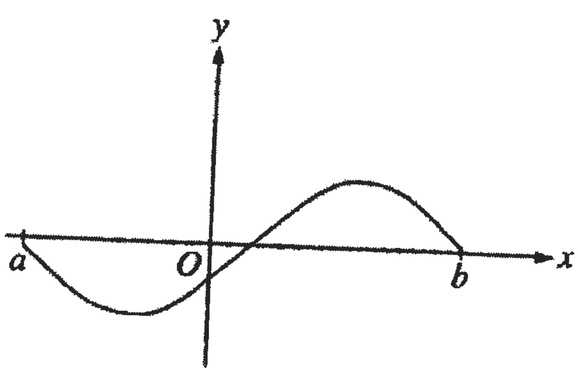

d.




b.




e.




c.

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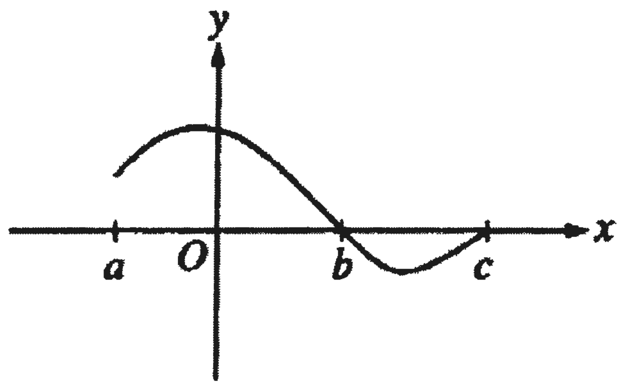
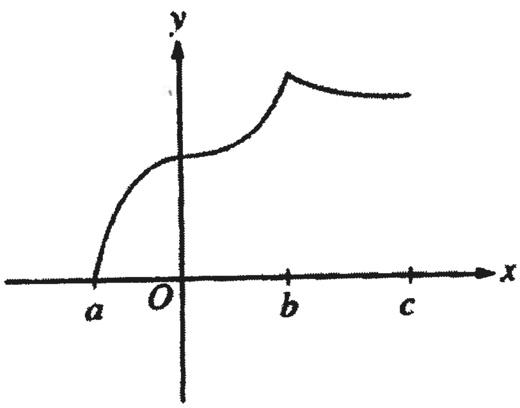
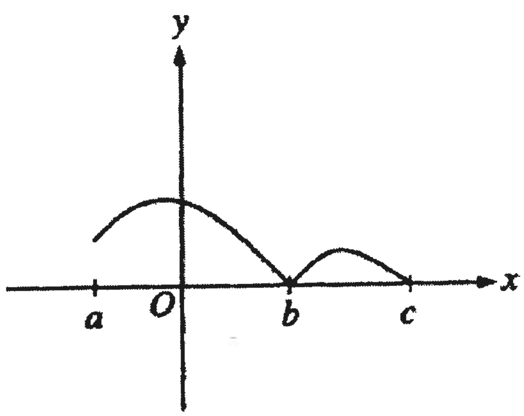
16.



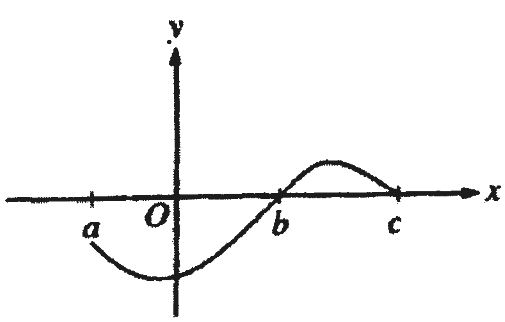

Let $f(x) = \int_a^x h(t)\,dt$, where $h$ has the graph shown above. Which of the following could be the graph of $f$?



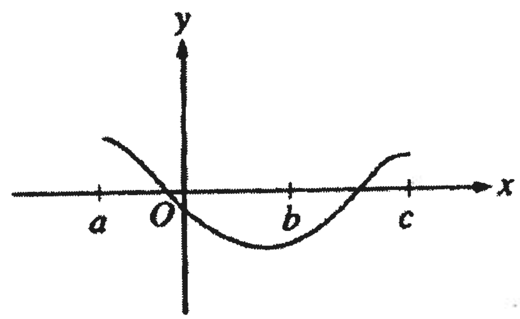

a.



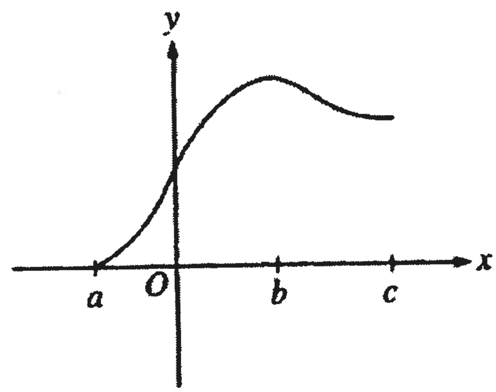

b.




c.




d.




e.